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Two Dimensional Anti-de Sitter Space and Discrete Light Cone Quantization

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Abstract

We realize the two dimensional anti-de Sitter (AdS_2) space as a Kaluza-Klein reduction of the AdS_3 space in the framework of the discrete light cone quantization (DLCQ). Introducing DLCQ coordinates which interpolate the original (unboosted) coordinates and the light cone coordinates, we discuss that AdS_2/CFT correspondence can be deduced from the AdS_3/CFT . In particular, we elaborate on the deformation of WZW model to obtain the boundary theory for the AdS_2 black hole. This enables us to derive the entropy of the AdS_2 black hole from that of the AdS_3 black hole.

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One of the main progresses achieved recently in the string theory is the *AdS/CFT* duality [1,2], which connects the gravity in the D -dimensional anti-de Sitter (AdS_D) space and the $(D - 1)$ dimensional conformal field theory (*CFT*) on its boundary. Among the *AdS/CFT* dualities in the various dimensions are the AdS_3/CFT and AdS_2/CFT dualities relevant for the black hole physics, since most of the black holes in the string theories are known to contain either AdS_3 space or AdS_2 space in their near horizon geometries [1,3]. Thus, the *AdS/CFT* dualities in low dimensions would play a key role in understanding the quantum aspects of the black holes. However, compared with the case of the AdS_3/CFT duality [4–6] the AdS_2/CFT duality is less well discussed in the literature. Observing that the near horizon geometry of the three dimensional BTZ (Bañados-Teitelboim-Zanelli) black hole [7] becomes effectively AdS_2 in the low energy regime, one may attempt to derive the AdS_2/CFT duality from the known AdS_3/CFT . This approach was taken by Strominger in his recent work on AdS_2/CFT duality [8].

Here in this paper we will employ a different strategy to derive the AdS_2/CFT duality, namely the DLCQ (discrete light cone quantization) [9], which reveals the relationship between two dualities more transparently. If the BTZ black hole is viewed in the light cone frame along the circle direction, the metric components in the light like directions are constant and can be scaled by boosting the frame. Thus, if the light cone coordinate, x^- is taken to be periodic, the Kaluza-Klein compactification can be easily performed. In order to have a periodic light cone coordinate, we employ the DLCQ procedure, which has been discussed recently [10] in the context of the Matrix M-theory [11]. It is found useful to introduce DLCQ coordinates, which interpolate the original (unboosted) coordinates and the light cone ones when we apply the DLCQ procedure to the AdS_3 black hole. One advantage of this approach is that we do not need to confine ourselves to the near horizon region.

Let us begin with the well-known $D1$ - $D5$ black hole in ten dimensions, which has its near horizon geometry as $M_{BTZ} \times S^3 \times T^4$ [12]

$$\frac{ds^2}{\alpha'} = \frac{U^2}{l^2}(-dt^2 + dx_5^2) + \frac{U_0^2}{l^2}(\cosh \sigma dt + \sinh \sigma dx_5)^2$$

$$+ \frac{l^2}{U^2 - U_0^2} dU^2 + l^2 d\Omega_3 + \frac{r_1}{r_5} \sum_{i=6}^9 \frac{(dx^i)^2}{\alpha'}, \quad (1)$$

where $x_5 \sim x_5 + 2\pi R_s$, $x_{6,7,8,9} \sim x_{6,7,8,9} + 2\pi V^{\frac{1}{4}} \alpha'^{\frac{1}{2}}$. Hereafter we will take $\alpha' = 1$ for the sake of convenience. M_{BTZ} corresponds to the well known three dimensional BTZ black hole, which has mass and angular momentum as follows

$$Ml = \frac{1}{8Gl}(\rho_+^2 + \rho_-^2) = \frac{R_s^2}{8Gl^3} U_0^2 \cosh 2\sigma, \quad (2)$$

$$|J| = 2\frac{1}{8Gl}\rho_+\rho_- = \frac{R_s^2}{8Gl^3} U_0^2 |\sinh 2\sigma|. \quad (3)$$

Here we note that the BTZ black hole becomes BPS object as we take the limit $\sigma \rightarrow \pm\infty$ while keeping M, J finite by making $U_0 \rightarrow 0$; in this limit $M \mp J/l \rightarrow 0$.

It is tempting to reduce the three dimensional black hole to a two dimensional AdS black hole by the Kaluza-Klein (KK) reduction. The early attempt was made in ref. [13], where the four dimensional extremal Reissner-Nordström black hole is described as $AdS_2 \times S^2$: The BTZ black hole can be viewed as a two dimensional dilatonic AdS black hole with a $U(1)$ charge, when we apply the KK reduction along the spatial S^1 direction to the BTZ black hole. A more elaborated description of AdS_2 is given by Strominger in his recent work [8], by taking the very near horizon limit of the extremal black hole.

In this paper we take the KK reduction along nearly lightlike circle rather than along the spatial circle. We first give naive idea on this. The metric of the BTZ black hole in the light cone frame becomes

$$ds^2 = \frac{U_0^2 e^{2\sigma}}{2l^2} dx^{+2} + \frac{U_0^2 e^{-2\sigma}}{2l^2} dx^{-2} + \frac{2U^2 - U_0^2}{l^2} dx^+ dx^- + \frac{l^2}{U^2 - U_0^2} dU^2, \quad (4)$$

where $x^\pm = (x^5 \pm t)/\sqrt{2}$. It suggests that the light cone coordinate, x^- may be chosen to be compactified, since the dilaton is constant. Here we need to employ the DLCQ procedure in order to have a periodic light like coordinate. The DLCQ procedure has been recently discussed [10] in the context of the Matrix M-theory [11]. So some part of the analysis to be presented also will be useful to study the Matrix M-theory. Let us suppose that x^5 is

periodic, $x^5 \sim x^5 + 2\pi R_s$, i.e., $(x^+, x^-) \sim (x^+ + \sqrt{2}\pi R_s, x^- + \sqrt{2}\pi R_s)$. Then by a Lorentz boost, we have $(x'^+, x'^-) \sim (x'^+ + \sqrt{2}\pi R_s e^\alpha, x'^- + \sqrt{2}\pi R_s e^{-\alpha})$, $R_s/R = (\cosh 2\alpha)^{-\frac{1}{2}}$, where $x'^\pm = e^{\pm\alpha} x^\pm$ are the boosted light cone coordinates. In the limit of the large boosting, i.e. when the boosting parameter $\alpha \rightarrow -\infty$, (equivalently $R_s \rightarrow 0$ with R kept finite), x'^- becomes periodic; $(x'^+, x'^-) \sim (x'^+, x'^- + 2\pi R)$.

The metric reads in terms of the boosted coordinates as,

$$ds^2 = -\frac{2U^2(U^2 - U_0^2)}{U_0^2 l^2} e^{2(\sigma-\alpha)} (dx'^+)^2 + \frac{l^2}{U^2 - U_0^2} dU^2 + \frac{U_0^2 e^{-2(\sigma-\alpha)}}{2l^2} \left(dx'^- + \left(2\frac{U^2}{U_0^2} - 1 \right) e^{2(\sigma-\alpha)} dx'^+ \right)^2. \quad (5)$$

Here we observe that the dilaton factor, i.e., the compactification radius is constant and x'^+ plays the role of time coordinate. In the DLCQ limit, the geometry becomes $AdS_2 \times S^1$.

However in order to be more transparent on the deformation of a spatial circle to make a nearly light like circle, we introduce new coordinates (\hat{t}, U, \hat{x}) , called DLCQ coordinates, which interpolate the original coordinates (t, U, x^5) (when $\alpha = 0$) and the infinitely boosted light cone coordinates (x'^+, U, x'^-) (when $\alpha \rightarrow -\infty$): $(\hat{t}, \hat{x}) \sim (\hat{t}, \hat{x} + 2\pi R_s \sqrt{\cosh 2\alpha})$,

$$\hat{x} = \frac{e^\alpha x'^+ + e^{-\alpha} x'^-}{\sqrt{e^{2\alpha} + e^{-2\alpha}}}, \quad \hat{t} = \frac{e^{-\alpha} x'^+ - e^\alpha x'^-}{\sqrt{e^{2\alpha} + e^{-2\alpha}}}. \quad (6)$$

In the boosted frame \hat{x} is identified as a periodic coordinate and the time coordinate \hat{t} is chosen such that $\partial_{\hat{t}}$ is orthogonal to $\partial_{\hat{x}}$. In the infinite boosting limit ($\alpha \rightarrow -\infty$), $\hat{t} \rightarrow x'^+ = e^\alpha x^+$ and $\hat{x} \rightarrow x'^- = e^{-\alpha} x^-$.

In this DLCQ coordinates, the metric reads as

$$ds^2 = \frac{U^2 + U_0^2 \sinh^2(\sigma' + \alpha)}{l^2 \cosh 2\alpha} \left(d\hat{x} + \frac{U_0^2 \sinh 2\sigma' - (2U^2 - U_0^2) \sinh 2\alpha}{2(U^2 + U_0^2 \sinh^2(\sigma' + \alpha))} d\hat{t} \right)^2 - \frac{U^2(U^2 - U_0^2) \cosh 2\alpha}{l^2(U^2 + U_0^2 \sinh^2(\sigma' + \alpha))} d\hat{t}^2 + \frac{l^2}{U^2 - U_0^2} dU^2. \quad (7)$$

Now it becomes clear that we should keep $\sigma' = \sigma - \alpha$ finite in order to obtain the metric for the space $AdS_2 \times S^1$ in the limit, $\alpha \rightarrow -\infty$,

$$ds^2 = -\frac{2U^2(U^2 - U_0^2)}{U_0^2 l^2} e^{2\sigma'} d\hat{t}^2 + \frac{l^2}{U^2 - U_0^2} dU^2 + \frac{U_0^2 e^{-2\sigma'}}{2l^2} \left(d\hat{x} + a_{\hat{t}} d\hat{t} \right)^2, \quad (8)$$

where $a_{\hat{t}} = \left(2\frac{U^2}{U_0^2} - 1\right) e^{2\sigma'}$. It depicts an AdS_2 black hole with $U(1)$ charge and coincides with the metric Eq.(5) obtained by the naive DLCQ procedure. The mass and angular momentum of the black hole are given as

$$Ml = -J = \frac{R^2 U_0^2 e^{-2\sigma'}}{8Gl^3}. \quad (9)$$

Thus, the AdS_2 black hole can be described in terms of the extremal AdS_3 black hole in the framework of DLCQ. In passing we note that the way the DLCQ procedure results in the extremal limit is different from the usual one; $\sigma \rightarrow -\infty$, and $R_s \rightarrow 0$ so that $R e^{-\sigma'} = R_s \sqrt{\cosh 2\alpha} e^{-\sigma+\alpha}$ is kept finite.

The DLCQ coordinates are more useful when we discuss the boundary conformal field theory. In ref. [5], one of the authors explicitly showed that the boundary conformal field theory is given by a $SL(2, R) \otimes SL(2, R)$ WZW model, which is equivalent to the three dimensional gravity on AdS_3 , resorting to the Faddeev-Shatashvili procedure [14]. Since AdS_2 space is obtained by the Kaluza-Klein reduction from AdS_3 , the same procedure would lead us to the boundary conformal theory corresponding to the AdS_2 space. We first give the DLCQ reduction of the bulk Chern-Simons action [15], which may be rewritten as

$$I_{CS} = \frac{k}{4\pi} \int_M \text{tr} \epsilon^{\mu\nu} (A_{\hat{x}} F_{\mu\nu} - A_{\mu} \partial_{\hat{x}} A_{\nu}) + \frac{k}{4\pi} \int_{\partial M} \text{tr} A_{\hat{t}} A_{\hat{x}} \\ - \frac{k}{4\pi} \int_M \text{tr} \epsilon^{\mu\nu} (\bar{A}_{\hat{x}} \bar{F}_{\mu\nu} - \bar{A}_{\mu} \partial_{\hat{x}} \bar{A}_{\nu}) - \frac{k}{4\pi} \int_{\partial M} \text{tr} \bar{A}_{\hat{t}} \bar{A}_{\hat{x}},$$

where $A = (\omega + \frac{\epsilon}{l})_I^A dx^I J_A$, $\bar{A} = (\omega - \frac{\epsilon}{l})_I^A dx^I \bar{J}_A$, $[J_A, J_B] = \epsilon_{AB}^C J_C$, $[\bar{J}_A, \bar{J}_B] = \epsilon_{AB}^C \bar{J}_C$, $\mu, \nu, \rho \in \{\hat{t}, U\}$, $A, B, C \in \{0, 1, 2\}$, and $I, J, K \in \{\hat{t}, U, \hat{x}^5\}$. We may cast the dreibein and the spin connection into the form of Kaluza-Klein ansatz as

$$e_I^A = \begin{pmatrix} e_{\mu}^a & -l e^{\psi} a_{\mu} \\ 0 & -\frac{l}{R} e^{\psi} \end{pmatrix}, \quad \omega_I^A = \begin{pmatrix} \omega_{\mu}^a & \omega_{\mu}^2 \\ \omega_{\hat{x}}^a & \omega_{\hat{x}}^2 \end{pmatrix},$$

where $a \in \{0, 1\}$. Then, imposing the torsion free conditions, which are obtained as part of the equations of motion we find that the Chern-Simons action reduces to the action for the two dimensional gravity as expected

$$I_{2D} = \frac{k}{2} \int \sqrt{-g} \left(e^{\psi} \left(R + \frac{2}{l^2} \right) - \frac{e^{3\psi} l^2}{4} f_{\mu\nu} f^{\mu\nu} \right), \quad (10)$$

where $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$.

From the metric for the BTZ black hole in the light cone frame Eq.(4), we obtain the black hole solution in terms of the gauge fields as follows.

$$\begin{aligned}
A^0 &= 0, \quad A^1 = 0, \\
A^2 &= \frac{U_0 e^{-\sigma'}}{l^2} \frac{-e^\alpha d\hat{t} + e^{-\alpha} d\hat{x}}{\sqrt{\cosh 2\alpha}}, \\
\bar{A}^0 &= -2 \frac{U e^{\sigma'}}{U_0 l^2} (U^2 - U_0^2)^{\frac{1}{2}} \frac{e^{-\alpha} d\hat{t} + e^\alpha d\hat{x}}{\sqrt{\cosh 2\alpha}}, \\
\bar{A}^1 &= -2 (U^2 - U_0^2)^{-\frac{1}{2}} dU \\
\bar{A}^2 &= -2 \frac{e^{\sigma'}}{U_0 l^2} \left(U^2 - \frac{U_0^2}{2} \right) \frac{e^{-\alpha} d\hat{t} + e^\alpha d\hat{x}}{\sqrt{\cosh 2\alpha}}.
\end{aligned} \tag{11}$$

Among those, only the components $A_{\hat{x}}$ and $\bar{A}_{\hat{x}}$ are physically important because the nontrivial geometrical structure of the three dimensional space is completely encoded by holonomies or Wilson loops of the Chern-Simons gauge fields [16], $W[C] = \mathcal{P} \exp(\oint_C A_{\hat{x}} d\hat{x})$, $\bar{W}[C] = \mathcal{P} \exp(\oint_C \bar{A}_{\hat{x}} d\hat{x})$, where C is a closed curve and \mathcal{P} denotes a path ordered product. Since in the limit where $\alpha \rightarrow -\infty$, $A_{\hat{x}} \rightarrow \sqrt{2} \frac{U_0 e^{-\sigma'}}{l^2}$, $\bar{A}_{\hat{x}} \rightarrow 0$, we see that the the right $SL(2, R)$ sector becomes trivial while the left $SL(2, R)$ sector remains relevant. It is consistent with the observation that the DLCQ and scaling procedure results in the BPS limit. This also explains how the isometry group of AdS_3 , $SL(2, R) \otimes SL(2, R)$ reduces to the isometry group of AdS_2 , $SL(2, R)$ in the framework of DLCQ.

Rewriting the boundary action obtained in ref. [5] in terms of the DLCQ coordinates, we find that the boundary theory for AdS_2 may be given by the WZW model, deformed by the DLCQ procedure. The total action is composed of the followings;

$$\begin{aligned}
I &= \frac{k}{4\pi} \int_M \text{tr} \epsilon^{ij} \partial_i A_j + \frac{k}{4\pi} \int_M \text{tr} \epsilon^{ij} A_i F_{ij} \\
&\quad - \frac{k}{4\pi} \int_{\partial M} \text{tr} \left[e^{2\alpha} (A_{\hat{x}} + \partial_{\hat{x}} g g^{-1})^2 + 2A_{\hat{t}} (A_{\hat{x}} + \partial_{\hat{x}} g g^{-1}) \right] \\
&\quad - \frac{k}{4\pi} \int_{\partial M} \text{tr} \partial_{\hat{t}} g g^{-1} \partial_{\hat{x}} g g^{-1} - \frac{k}{12\pi} \int_M \text{tr} (g^{-1} dg)^3, \\
\bar{I} &= -\frac{k}{4\pi} \int_M \text{tr} \epsilon^{ij} \partial_i \bar{A}_j - \frac{k}{4\pi} \int_M \text{tr} \epsilon^{ij} \bar{A}_i \bar{F}_{ij} \\
&\quad - \frac{k}{4\pi} \int_{\partial M} \text{tr} \left[e^{-2\alpha} (\bar{A}_{\hat{x}} + \partial_{\hat{x}} \bar{g} \bar{g}^{-1})^2 - 2\bar{A}_{\hat{t}} (\bar{A}_{\hat{x}} + \partial_{\hat{x}} \bar{g} \bar{g}^{-1}) \right]
\end{aligned}$$

$$+ \frac{k}{4\pi} \int_{\partial M} \text{tr} \partial_i \bar{g} \bar{g}^{-1} \partial_{\hat{x}} \bar{g} \bar{g}^{-1} + \frac{k}{12\pi} \int_M \text{tr} (\bar{g}^{-1} d\bar{g})^3, \quad (12)$$

where $i, j \in (U, \hat{x})$. With $(\tau, \theta) = \frac{1}{R}(e^{2\alpha}\hat{t}, \hat{x})$ we may identify I as the left sector of the action for the BTZ black hole given in ref. [5]. We also find that \bar{I} can be understood as the right sector of the action for the BTZ black hole in terms of $(\tau', \theta) = \frac{1}{R}(e^{-2\alpha}\hat{t}, \hat{x})$. The right sector is confined in the extreme low energy regime, thus, suppressed. The left $SL(2, R)$ sector only becomes relevant to the AdS_2 .

The DLCQ procedure presented here illustrates explicitly how AdS_2/CFT correspondence can be deduced from the AdS_3/CFT . It would be interesting to explore its consequences in various contexts. One may find an application of the DLCQ procedure readily in evaluation of the entropy for the AdS_2 black hole. To evaluate the entropy, we adopt the result of ref. [5], where the entropy of the three dimensional BTZ black hole is evaluated in accord with the proposal of Strominger [17]. Comparing the boundary action for the AdS_2 (12) with that for the BTZ black hole, given in ref. [5], we find that the expectation values of the Virasoro generators L_0 and \bar{L}_0 for the AdS_2 black hole are given by

$$\begin{aligned} n_L = \langle L_0 \rangle &= \frac{k U_0^2 R^2 e^{-2\sigma'}}{2l^4}, \\ n_R = \langle \bar{L}_0 \rangle &= \frac{k U_0^2 R^2 e^{2\sigma'}}{2l^4} e^{4\alpha}. \end{aligned} \quad (13)$$

Here we assume that $n_L = n_R = 0$ for an appropriately chosen vacuum state of the black hole. Then it follows from the Cardy formula that the entropy for the AdS_2 black hole is evaluated as

$$S = \sqrt{2\pi} k \frac{R U_0 e^{-\sigma'}}{l^2} + \sqrt{2\pi} k \frac{R U_0 e^{\sigma'}}{l^2} e^{2\alpha}. \quad (14)$$

As $\alpha \rightarrow -\infty$, the right sector does not contribute to the entropy, $S \rightarrow 4\pi k \sqrt{GM}$. (Some difficulty in evaluating the entropy by applying the Cardy formula to the boundary conformal theory was pointed out by Carlip [18]. This difficulty may be resolved if we consider the space-time Virasoro algebra, regarding the boundary conformal theory as the worldsheet action for the string on AdS_3 [6]. It would be worth while to extend the work of ref. [5] along this direction.)

We conclude this paper with a few brief remarks. We show that the AdS_2 black hole can be described as a DLCQ limit of the AdS_3 black hole. The boundary theory for AdS_2 is found to be a $SL(2, R)$ WZW model defined on the light like coordinate. If we are concerned with the gravity on AdS_2 , the zero mode sector of the AdS_3 suffices in the framework of DLCQ. In accordance with it, the zero mode sector of the WZW model given by the action Eq.(12) defines the boundary theory, hence a $SL(2, R)$ quantum mechanics. The correspondence between the gravity on AdS_2 and the $SL(2, R)$ boundary conformal quantum mechanics will be discussed in detail elsewhere. We note that the entropy of the AdS_2 black hole evaluated as Eq.(14) is the same as that of the AdS_3 black hole in the original frame. Thus, the entropy is preserved by the DLCQ procedure.

The present paper may be extended along various directions. One of the direction would be the DLCQ reduction of the string on AdS_3 [6]. It would be interesting to see if the DLCQ reduction leads us to a point source on AdS_2 and the entropy of AdS_2 black hole can be given by the quantum mechanics. This may clarify some subtle issues associated with the entropy of black holes in AdS_3 and AdS_2 . The two dimensional black holes have been one of the important subjects in string theory and gravity. The present work may enable us to discuss the various aspects of the two dimensional black holes [19,20] from the viewpoint of the three dimensional black hole. According to the conjecture of AdS/CFT correspondence [1] the (2+1) dimensional gravity on AdS_3 is supposed to be equivalent to the boundary (1+1) dimensional conformal Yang-Mills theory on its boundary. The present work also suggests that the boundary conformal theory corresponding to the gravity on AdS_2 may be obtained by the DLCQ reduction of the (1+1) dimensional Yang-Mills theory. Work along this direction is now in progress [21]. The most important application of the present work may be found in the Matrix M-theory. DLCQ reduction of the M-brane configurations may shed some light on the Matrix M-theory. After completing the work, we found that reduction of AdS_3 to AdS_2 along the light like coordinate also has been suggested in the study of M-brane configurations [22].

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